

Effectiveness of the Two-Step Dynamic Demand Estimation model on large networks

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Abstract—In this paper, the authors present a Two-Step approach that sequentially adjusts generation and distribution values of the (dynamic) OD matrix. While the proposed methodology already provided excellent results for updating demand flows on a motorway, the aim of this paper is to validate this conclusion on a real network: Luxembourg City. This network represents the typical middle-sized European city in terms of network dimension. Moreover, Luxembourg City has the typical structure of a metropolitan area, composed of a city centre, ring, and suburb areas. An innovative element of this paper is to use mobile network data to create a time-dependent profile of the generated demand inside and outside the ring. To support the claim that the model is ready for practical implementation, it is interfaced with PTV Visum, one of the most widely adopted software tools for traffic analysis. Results of these experiments provide a solid empirical ground in order to further develop this model and to understand if its assumptions hold for urban scenarios.

Keywords—*o-d estimation; Two-Step optimisation; Quasi-dynamic assumption; bilevel optimisation; GSM data;*

I. INTRODUCTION

Dynamic Traffic Assignment (DTA) models represent the current state of the practice for managing transportation systems. To be able to make accurate predictions about the network condition or the effect of new traffic policies, these models require a good knowledge of the travel demand, which is usually represented in the form of an origin-destination (OD) matrix.

In order to generate this matrix, while traditional demand generation models combine survey data and statistical tools [1], [2], more recent approaches have done a significant progress into including new data sources, such as Call Detail Records (CDR), GSM data, sensing data and geospatial data [3], [4]. Although these works showed that big data can largely improve the overall quality of the result, the estimated demand matrix is at most a concise representation of the regular demand patterns. Unfortunately, since dynamics of traffic systems are complex and depend on partially predictable phenomena such as weather conditions, daily demand patterns can substantially differ from the regular ones, because of structural and random deviations [5].

These deviations can be corrected by using traffic measurements, such as loop detectors, to update the existing (a-

priori) OD matrix. This problem, which is known in literature as the Dynamic Demand Estimation Problem (called DDEP in this paper), searches for time-dependent OD demand matrices able to best fit measured data. It can be applied for both within-day (intra-period) and day-to-day (inter-period) dynamic frameworks [6], as well as for offline (medium-long term planning and design) and on-line (real-time management) [7]. While for a detailed overview, the interested reader can refer to [8], we limit our discussion to recent works related to the off-line DDEP.

Classical approaches solve two interconnected optimisation problems, according to a bi-level formulation: in the upper level, time-dependent OD matrices are corrected in order to replicate the observations, while the lower level relates OD with path and link flows [8]. However, the resulting optimisation problem is highly underdetermined [9], and provides an accurate prediction only when the ratio between unknown and known variables (OD flows and traffic measurements, respectively) is close to one. From the modelling point of view, the easiest solution is to formulate the optimisation problem in a different way, in order to reduce the number of variables. This can be done, for instance, by introducing a parametric representation of the demand, as proposed in [10], or performing a Principal Component Analysis (PCA) [11]. Recently, Cascetta et al. [12] introduced the so-called “*quasi-dynamic assumption*”, which assumes that the generated demand for a certain OD pair is time dependent, while its spatial distribution is constant. Under this assumption, as demonstrated in [12], the DDEP problem becomes less underdetermined and more likely to find more robust results. Nevertheless, the authors point out that the resulting matrix will be “*intrinsically biased*”, since this assumption introduces an “*intrinsic error*”. To solve this problem, Cantelmo et al. [13] introduced a Two-Step procedure, which separates the problem in two sub-optimization problems. Through this procedure, authors correct sequentially generations and distributions in the demand matrix. In essence, the first step exploits the *quasi-dynamic* assumption in order to perform a broad evaluation of the solutions space, while in the second step the estimated OD flows are further updated in order to reduce the intrinsic error.

From the data-driven point of view, the most widely adopted procedure is to include new data sources, such as measured speeds [14], link density [15] and route travel time [16], within the Objective Function (OF) to be minimised. As

expected, by increasing the number of knowns in the optimisation problem, and by including information on the actual route choice, the solution reliability largely increases.

Driven by these considerations, in this paper we implement the Two-Step approach, already presented in [13], to the network of Luxembourg, and we extend the goal function in order to include mobile network data within the DDEP. The contribution is twofold. On one hand, we show that the Two-Step approach outperforms the standard formation on a real-life network. To support the claim that the model is ready for practical implementation, it is interfaced with PTV Visum, one of the most widely adopted software tools for traffic analysis. The second contribution regards the mobile network data. While these data have been widely adopted for generating dynamic OD matrix [3], their use within the OD/route flow estimation is still limited [17]. The main reason is the low level of precision of this information, which makes the match between observations and road segments quite challenging. In the proposed work, mobile network data are used to directly estimate the time-dependent demand profile, thus no matching is required.

II. METHODOLOGY

A. MAMBA-DEV Matlab package

To be able to solve a DDEP on a large real-sized network, we developed a Matlab package for solving the DDEP using PTV Visum as DTA model (Demand Estimation for Visum - DEV).

The package allows performing assignment-free dynamic or static OD estimation, using a deterministic and/or stochastic approximation of the gradient [13]. The model also includes the Two-Step approach, which is presented in the next section. While the MAMBA-DEV package has been designed for Luxembourg City, it can work with any network.

B. Two-Step Approach

While for a detailed overview of this model we refer to [13], in this section we briefly present its main characteristics.

The standard DDEP, called “*Single-Step*” in this paper, is generally solved as an optimisation problem. Its formulation requires the specification of the OF, its variables and its constraints, which are related to feasibility and routing conditions. Considering different types of measures and by adopting an offline approach, the OF can be formulated as:

$$(\mathbf{d}_1^*, \dots, \mathbf{d}_n^*) = \arg \min \begin{bmatrix} z_1(\mathbf{l}_1, \dots, \mathbf{l}_{n'}, \hat{\mathbf{l}}_1, \dots, \hat{\mathbf{l}}_{n'}) \\ + z_2(\mathbf{n}_1, \dots, \mathbf{n}_{n'}, \hat{\mathbf{n}}_1, \dots, \hat{\mathbf{n}}_{n'}) \\ + z_3(\mathbf{x}_1, \dots, \mathbf{x}_{n'}, \hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_{n'}) \\ + z_4(\mathbf{r}_1, \dots, \mathbf{r}_{n'}, \hat{\mathbf{r}}_1, \dots, \hat{\mathbf{r}}_{n'}) \end{bmatrix} \quad (1)$$

Where $\mathbf{l}/\hat{\mathbf{l}}$ are the simulated values and the corresponding measurements on the links, $\mathbf{n}/\hat{\mathbf{n}}$ are the simulated values and the corresponding measurements on the nodes, $\mathbf{x}/\hat{\mathbf{x}}$ are the estimated values and a-priori information on the dynamic demand, $\mathbf{r}/\hat{\mathbf{r}}$ are the simulated values and the measurements on routes, \mathbf{d}_n^* is the estimated demand matrix

for time interval n and, finally, $z := \{z_1, z_2, z_3, z_4\}$ is the estimator of the deviations between the simulated/estimated and the corresponding measured/a-priori values. The consistency between simulated traffic performances and the estimated demand is obtained directly by performing a dynamic traffic assignment (DTA).

The applicability of Equation (1) is general, but has its shortcomings. Among others, when dealing with a large number of variables, Equation (1) collapses to a local adjustment of the a-priori OD flows, rather than a real estimation. As discussed in the introduction, this is one of the main reasons for which introducing the quasi-dynamic assumption sounds reasonable. On the one hand, this introduces an approximation, while on the other it allows the algorithm to avoid local minima.

In the proposed Two-Step procedure, the first step aims at optimising the generation values of each zone in each time interval, while maintaining constant the dynamic trip distributions derived by the seed matrix. To achieve this goal, the objective function in (1) can be generally rewritten for the first step as:

$$(\mathbf{E}_1^*, \dots, \mathbf{E}_n^*) = \arg \min \begin{bmatrix} z_1(\mathbf{l}_1, \dots, \mathbf{l}_{n'}, \hat{\mathbf{l}}_1, \dots, \hat{\mathbf{l}}_{n'}) \\ + z_2(\mathbf{n}_1, \dots, \mathbf{n}_{n'}, \hat{\mathbf{n}}_1, \dots, \hat{\mathbf{n}}_{n'}) \\ + z_3(\mathbf{x}_1, \dots, \mathbf{x}_{n'}, \hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_{n'}) \\ + z_4(\mathbf{r}_1, \dots, \mathbf{r}_{n'}, \hat{\mathbf{r}}_1, \dots, \hat{\mathbf{r}}_{n'}) \end{bmatrix} \quad (2a)$$

S.t.

$$x_n^{OD} = E_n^O d_{DO}^{Seed, n} \quad \forall O, \forall D, \forall n \quad (2b)$$

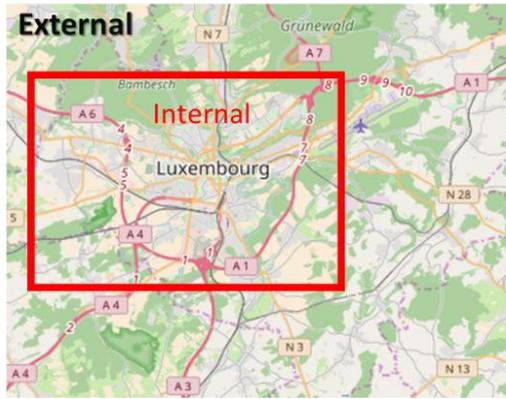
Where E_n^O is the generation of origin zone O and time interval n , \mathbf{E}_n^* is the generation vector containing generation from all origins in time interval n , X_n^* is the number of trips originated in O with destination D in time interval n and $d_{DO}^{Seed, n}$ is the matrix probability distribution between traffic zone D and traffic zone O in time interval n .

The goal of the first step is to act on the seed matrix in order to obtain a reasonable generation value before moving to the second step, in which the dynamic distributions are corrected according to (1) in order to reduce the intrinsic error.

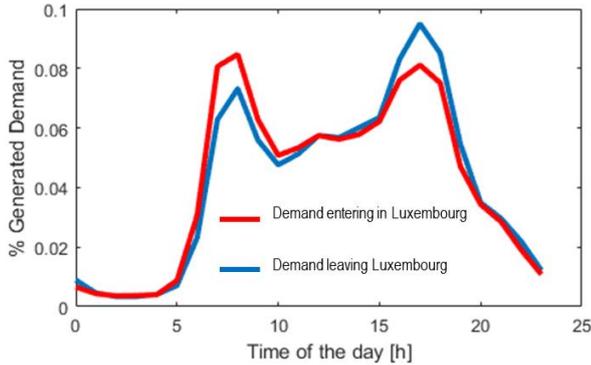
C. Including mobile network data in the Objective Function

As pointed out in the introduction, it is commonly accepted that including more information within the goal function leads to a more robust result for the DDEP. Clearly, this cannot be considered a general rule since, when different data sources are combined, the solution space of the OF can become more irregular. In this sense, mobile network technology, because of its spatial/temporal coverage and because of the great volume of information, seems an ideal data source for the DDEP. While the correlation between traffic demand and mobile data is well known [3], this source of information is hard to implement within the DDEP. When dealing with GPS information, one of the most critical elements is to match the

GPS coordinates and the road network. Mobile network data provide at most the geographic position at connected antenna level, so no direct road network match is possible. However, by clustering antennas located on the border of each traffic zone, it is possible to count active connections that are entering or exiting the zones (i.e. the number of *handovers*). Unfortunately, mobile network data cannot be considered as the sole source of information for the DDEP, as they are subject to intrinsic errors such as the split of the user base between multiple network operators and the degree of activity on the network as well as the general mobile penetration rates. In this work, we use aggregated handover counts between antennas of 2G, 3G and 4G radio technologies of Luxembourg mobile network operator POST Luxembourg. The data consists of the hourly counts of connections being handed off between pairs of antennas, thus respecting users' privacy.



(a)



(b)

Figure 1: (a) Internal and External antenna clusters for Luxembourg City; (b) Emission flow from and to Luxembourg City.

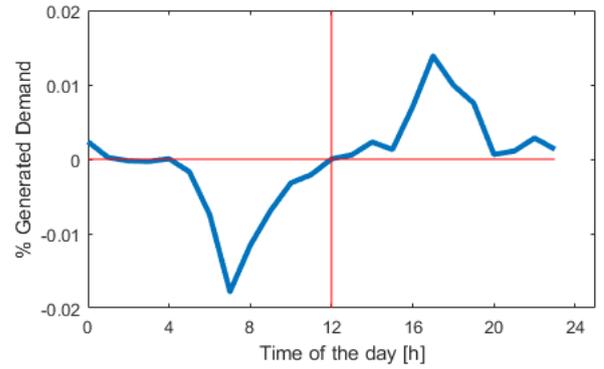
We propose the following two criteria to exploit demand emission flows estimated through the mobile network data:

- 1) Antenna clusters need to be large enough to minimise the “ping-pong” effect, i.e. repeatedly counting the same users ‘bouncing’ back and forth between two antennas;
- 2) Cluster edges shall be positioned so as to maximise the difference between number of people entering and leaving the study area;

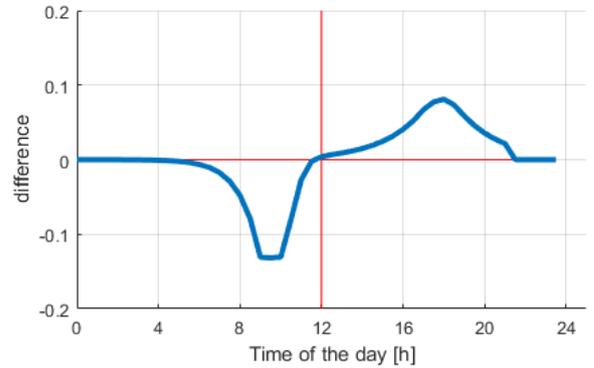
Since we are focusing on Luxembourg City, we created two different clusters. One cluster captures the trips generated from the city to the external zones, while the other one captures those entering Luxembourg City, as shown in Figure 1. This procedure can be easily extended to any urban area, in which mobile connection handovers can be used to calculate the flows exchanged between the study area and the external centroids. Although the profile showed in Figure 1b looks realistic, we do believe that to simply include the emission flows within the goal function may still lead to a biased estimation, since it is equivalent to over-imposing a certain time-dependent profile to the demand. Instead, we propose to use the difference between entering and exiting flow, as in Equation (3):

$$\Delta E_n^{GSM} = \frac{E_n^{GSM-IntZones}}{\sum_n E_n^{GSM-IntZones}} - \frac{E_n^{GSM-ExtZones}}{\sum_n E_n^{GSM-ExtZones}} \quad \forall n \quad (3)$$

Where $E_n^{GSM-IntZones}$ and $E_n^{GSM-ExtZones}$ are the mobile connection handovers to the internal and external zones, respectively. Figure 2 show the profile of ΔE_n^{GSM} for the real data (2a) and the a-priori OD matrix (2b).



(a)



(b)

Figure 2: (a) Profile obtained through the real-data; (b) Profile obtained through the 4-step approach.

As showed in Figure 2, the profile obtained by combining the classical Four-Step approach with a departure time choice model (2b) is comparable to the one obtained with

the mobile network data (2a). We can also identify quite easily the two errors within the a-priori OD matrix. First, the average departure time for the morning peak is wrongly shifted in time. Second, there is a difference in the scale, on the y-axis. The reason is that, in this application, we calculate the OD flows for the morning and evening commute, thus the demand in the afternoon is highly underestimated. This suggests that, by including Equation 3 within the OF of the DDEP, we can use mobile network data as a soft constraint to correct the demand obtained through classical demand generation models.

III. CASE STUDY

Synthetic experiments have been conducted on the urban network of Luxembourg City (Figure 3). While real traffic measures are available in Luxembourg, authors believe that assessing the quality of the proposed algorithm in a controlled experiment is a fundamental step before moving to the practical implementation.



Figure 3: Network of Luxembourg City, Luxembourg.

The network, which consists of 2744 active links, 1480 nodes and 17 traffic zones, represents the typical middle-sized European city in terms of network dimension. Moreover, Luxembourg City has the typical structure of a metropolitan area, composed of the city centre, ring, and suburb areas. OD flows are estimated over 24 hours assuming a 30-minutes departure interval. Under this assumption, the dynamic matrix contains 13872 variables to be estimated. The real matrix amounts to 239.966 trips, and with such an amount no congestion is expected on the network. Simulated measures for this network are available on a total of 32 counting sections – the links containing these sections are shown in red. Finally, the a-priori OD matrix, hereafter simply called *Seed* matrix, amounts to 171.060 trips, thus it significantly underestimates the number of trips in the network.

The DDEP is solved using both the Single-Step (SS) and Two-Step (TS) approaches. In both cases, the well-established Simultaneous Perturbation Stochastic Approximation (SPSA) is the numerical solution method adopted for the optimisation. In order to reduce the computational time, we adopted the one-sided version of this model. The interested reader can refer to [13] for more details on the solution algorithm. Similarly, we performed two different sets of experiments:

Scenario 1: Only traffic counts are included within the OF.

Scenario 2: Traffic counts and mobile data are included within the OF.

Finally, the Root Mean Square Error (RMSE) metric is the estimator adopted to quantify the error.

A. Scenario I: Only traffic counts

We opted for an uncongested scenario to primarily assess the capability of the model in handling a large number of variables, while at the same time considering a smooth goal function. The gradient is calculated as the average of 300 stochastic perturbations of the current matrix for the SS and 100 stochastic perturbations for the TS model. As shown in Figure 4, results confirm that, when the number of variables is large, SS model performs a quite local adjustment of the OD demand. Specifically, to obtain a reliable estimation of the gradient, the number of stochastic perturbations should be approximately 10% of the number of variables [18]. This entails 1382 DTA simulations for each iteration (~46 hours).

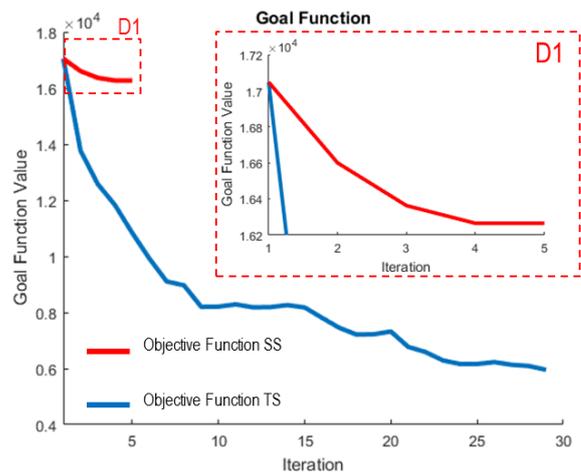


Figure 4: Goal Function trend

By contrast, introducing the strict quasi-dynamic assumption, there are only 816 variables to update. As a consequence, even with fewer replications of the gradient, the estimation is more robust, and the improvement is network wide. This is shown in Figure 5, where the scatter of the link flows is presented, and in Table 1. Specifically, in Table 1 we reported the error with respect to real link speed and real OD flows, which are used for validation purposes.

TABLE I.

	<i>Seed</i>	<i>Two-Step</i>	<i>Single Step</i>
RMSE Speed (Km/h)	3.73	2.47	3.66
RMSE OD (Veh/h)	42.25	37	43.00

It should be pointed out that, while the RMSE of the *Seed* may be considered low, we are considering in this experiment only morning and evening commute, thus a large number of OD/measures during the off-peak hours present a low error.

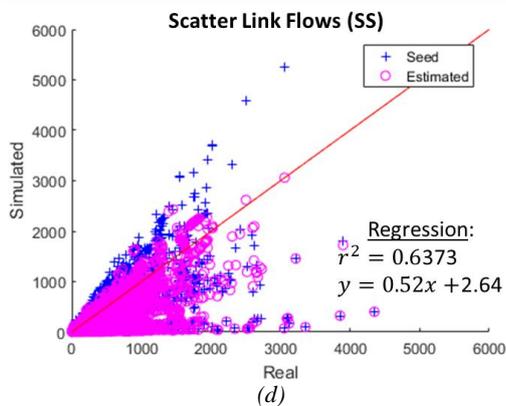
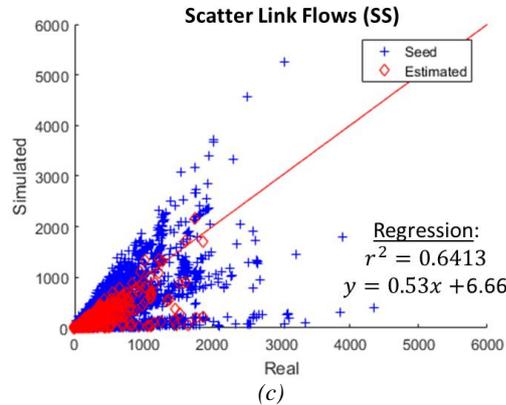
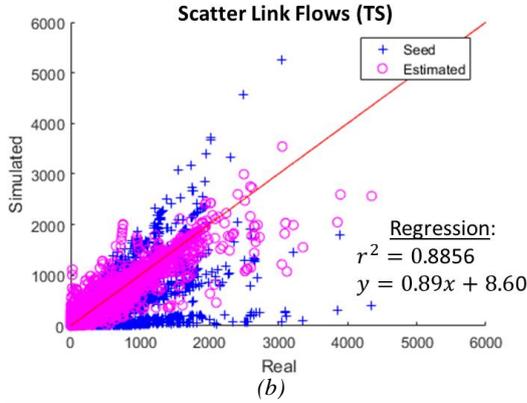
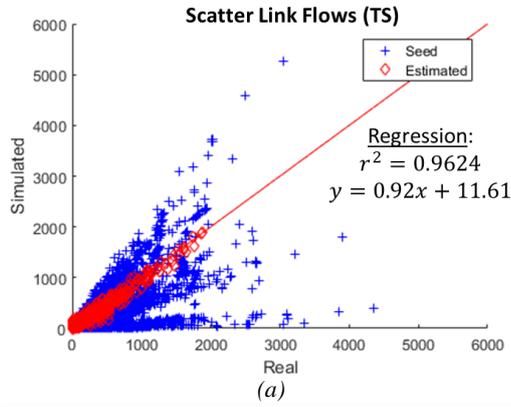


Figure 5: (a) Simulated vs Real link flows on the detector for the Two-Step; (b) Simulated vs Real link flows on all links for the Two-Step; (c) Simulated vs Real link flows on the detector for the Single Step; (d) Simulated vs Real link flows on all links for Single Step;

However, the resulting traffic pattern during the rush hour is substantially wrong, as shown in Figure 5, where we can clearly see that the *Seed* demand matrix is both overestimating and underestimating link flows.

Finally, results in Table 1 provide another important insight on the quality of the results. The SS model not only performed a local adjustment of the link flows but also increased the error with respect to the real matrix. By contrast, the proposed model is reducing the error according to all the performance measures.

B. Scenario II: Including mobile network data

In this subsection, we show the improvement related to using the mobile network data data within the goal function. In this case, the synthetic profile illustrated in Figure 2b has been used to simulate the mobile network data for the synthetic experiment. Results of this experiment are quite unexpected.

TABLE II.

	<i>Seed</i>	<i>TS</i>	<i>SS</i>
RMSE Speed (Km/h)	3.73	2.98	3.66
RMSE OD (Veh/h)	42.25	40.01	66.02

As showed in Table 2, the error in terms of OD flows is, at the end of the estimation, larger than in the previous case, showing that, for this uncongested network, the TS approach manages to find a better solution without the mobile data. However, as reported in Figure 6, when mobile data are included within the OF, the number of iterations required for solving the DDEP strongly decreases.

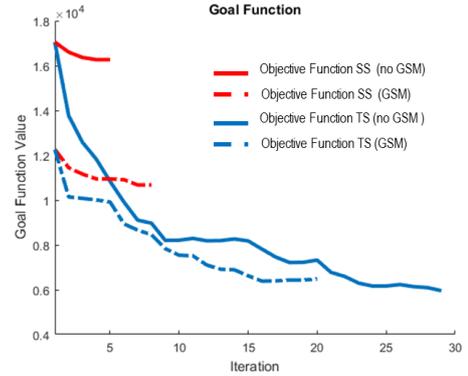


Figure 4: Goal Function trend, with and without GSM data

As predictable, the same property is not observed for the SS model, which simply collapses on the closest local minima. However, when this model is combined with the mobile network data, the error on the link flows decreases with respect to the base case presented in Scenario I (the RMSE is 3% lower).

A second and fundamental result concerns the stability of the estimated matrices. The RMSE of the OD flows between the solution of Scenario I and II are 27 and 48 veh/h for the TS and SS model, respectively. Although the Single-Step model

has a small OF improvement, the distance between the two estimated matrices is twice the distance of those estimated through the Two-Step approach. This means that the Two-Step approach not only manages to have a larger OF improvement but also to provide more reliable results. These findings are in line with the conclusions already presented in [13]. In general, we can claim that, since the two steps approach sequentially reduces the dimension of the solution space while keeping a lower number of variables with respect to the conventional Single-Step approach, it will provide a more reliable estimation [9].

IV. CONCLUSIONS AND FUTURE RESEARCH

The motivation for the research conducted in this paper is twofold. First, we aimed to generalise the effectiveness of a two-step approach for the Dynamic Demand Estimation Problem already introduced in [13] for a general urban network. Second, we performed a systematic assessment for the network of Luxembourg City, a fundamental step in order to use the proposed methodology for real applications. More specifically, the proposed Two-Step approach is a simple procedure to iteratively reduce the solution space without increasing the problem complexity. Results presented in this paper suggest that this methodology is suited for improving the reliability of the estimated travel demand and for performing a broader analysis of the solution space with respect to the conventional approach. While this model has some similarity with the quasi-dynamic approach proposed by Cascetta et al. [12], by performing a double-optimization, it also manages to overcome limitations related to the so-called “intrinsic error” of the quasi-dynamic assumption.

From a practical point of view, the proposed model has been implemented within the MAMBA-DEV Matlab package for the OD estimation, which exploits PTV Visum as traffic assignment module. Thus, the proposed model can be easily implemented with other networks, and we can conclude that the model is ready for practice. On this point, authors incorporated mobile network data as a soft constraint within the objective function, showing that this information largely increases the convergence speed.

Straightforward steps to future work are (i) validating the proposed results for a congested network and (ii) using the real data for performing the Dynamic Demand Estimation on Luxembourg City. More long-term objectives are to further extend MAMBA-DEV, in order to account for a larger set of models, including algorithms suited for solving on-line estimation and prediction problems.

ACKNOWLEDGMENT

The authors acknowledge for financing grant: AFR-PhD grant 6947587 IDEAS (Fonds National de Recherche FNR) and FNR Core Project C13/IS/5825301 MAMBA. The authors would also like to thank POST Luxembourg for providing the mobile network data.

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