Age of Information in IEEE 802.11p

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***Abstract*—Vehicle-to-Everything (V2X) communication is essential for facilitating connected and automated driving, where vehicles and other road traffic participants share data instantaneously and cooperate to solve tricky traffic situations in milliseconds. This paper proposes two stochastic models for the V2X standard IEEE 802.11p to characterize amongst other things the Age of Information (AoI), a recently-proposed metric that measures the freshness of information. The first model is based on renewal process analysis of a tagged station with mean field approximation, while the second one adopts Markov chain approach with network level view. Analytical results show that IEEE 802.11p, given its adaptability to event-triggered and aperiodic messaging, supports advanced cooperative driving scenarios.**

***Index Terms*—Age of Information, MAC Access Delay, CSMA Networks, Vehicular Networks, V2X Communications, Full Connectivity.**

I. Introduction

In today's vehicles, there are several line-of-sight sensors including radar and cameras, which enable features such as adaptive cruise control and lane keeping support. However, these sensors are unable to see beyond physical barriers. They see what the human eye spots but can react much faster to sudden changes compared to a human driver. Furthermore, line-of-sight sensors can detect objects in the immediate vicinity but they have difficulty predicting the intentions of detected objects. The Vehicle-to-Everything (V2X) wireless sensor addresses the shortcomings of line-of-sight sensors by charting both location and intention of other vehicles, and it has the ability to see beyond other objects in real-time (with updates provided in a matter of milliseconds). V2X communication is essential for connected and automated driving since it closes the gap between long-range cellular technology and line-of-sight sensors by providing information that beats the reaction time of the former and the range of the latter.

One of the key technologies designed for V2X communications is IEEE 802.11p, called Dedicated Short-Range Communication (DSRC) in the US and ITS-G5 in Europe. This technology, which is an evolution of WiFi, allows for instant communication in the vehicular environment without the need of establishing a network (communication takes place outside the context of a Basic Service Set (BSS)). IEEE 802.11p uses Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) to schedule transmissions on the shared communication channel. The basic communication mode in vehicular ad hoc networks is one-hop broadcast and messages for supporting cooperative driving are Cooperative Awareness Message (CAM), Collective Perception Message (CPM), and Maneuver Coordination Message (MCM) [1], [2].

Since the CSMA protocol is used for exchanging real-time control information, proper ways to characterize its performance are required. Traditional metrics, such as throughput and average delay, do not provide information about the "freshness" of control data available at the network nodes. The recently suggested Age of Information (AoI) metric [3]–[5] addresses this limitation. AoI is a key metric for systems where messages are generated by information sources that send updates frequently, which receiving nodes are dependent upon. Our objective is to analyze the AoI for broadcast transmissions in IEEE 802.11p, where updates are generated by each node and directed to all other neighboring nodes in the vicinity. Each node maintains a database collecting the most recent information it received from any other node around. To this end, the main contribution of this paper are the following two system models:

1) *Node model*, which is based on a mean field analysis, obtained by taking the point of view of a tagged node;
2) *Network model*, which is based on a bi-dimensional Discrete Time Markov Chain (DTMC) from [6], whose state description looks at the wireless channel.

Through comparison with simulations it turns out that both models are accurate, while providing different simplified versions of the full system state. The Network model accounts for nodes being backlogged or idle and for the outcome of channel usage. This state information is tracked by means of a bi-dimensional Markov chain. The Node model is simpler, since it only keeps track of the backlogged state of a single tagged node and summarizes the effect of all other nodes on the shared channel as a "mean field".

There is a vast amount of literature on the IEEE 802.11 Medium Access Control (MAC) analytical models. Most often the well known CSMA/CA throughput analysis grounded on seminal Bianchi's model [7] is generalized. For example, MAC access delay analysis is presented in [8], while the case of non saturated nodes is considered in [9], [10]. Underlying modeling assumptions are systematically analyzed and validated in [11]. There are also many relevant models of broadcasting in IEEE 802.11p, e.g. [12], [13].

More recent related work is dedicated to the behavior of the CSMA protocol in V2X scenarios, e.g., highway [14], [15] or urban intersection [16]. Finally, AoI for CSMA networks is analysed in [17]. We focus on the AoI in IEEE 802.11p network using broadcast transmissions. Our work is a continuation of [18], [19] for the full connectivity case. It brings some interesting insight into the understanding of the dynamics of a CSMA network.

The rest of the paper is organized as follows. Section II sets out the modeling stage, by giving a concise recap of the wireless channel access procedure, outlining the modeling assumptions and presenting the notations. The two proposed analytical models are presented in Section III and Section IV respectively. Performance results are introduced in Section V. A summary and an outlook to further work are given in Section VI.

II. System model

A. Discussion of assumptions

We consider a set of n nodes exchanging one-hop broadcast messages. The nodes use a non-persistent CSMA protocol to share the wireless channel. The following assumptions are made for both Network and Node models:

1) Full-connectivity (no hidden nodes). All nodes sense each other and can decode messages successfully from any other node, provided there is no overlap of multiple transmissions, i.e., a collision.
2) New messages arrive at nodes according to a Poisson process of mean rate λ (or Bernoulli process with the same mean rate).
3) Message transmission time is same for all nodes.

The first assumption is essential to state the models presented in this paper. It leads to models useful to study a vast array of practical systems, e.g., a platoon of vehicles or a swarm of drones. The above are characterized by a relatively stable and compact network topology that makes it feasible not to incorporate hidden nodes and relative nodes' mobility into the analysis.

The other two assumptions can be relaxed, though at the expense of making the stated models quite cumbersome and possibly losing the insight offered by simpler mathematics. It is anyhow interesting to mention the extension possibilities.

The second assumption listed above can be relaxed by considering more versatile arrival processes, e.g., a phase-type arrival process, or a Markovian Arrival Process. Although mainly for modeling purposes, Poisson arrivals (or their discrete version – Bernoulli arrivals) could be an interesting approximation in view of the following facts: (i) it leads to conservative performance with respect to constant message generation intervals; (ii) measurements of message generation in V2X shows that it is not strictly periodic, rather it exhibits some variability [20].

It is possible to extend last assumption, i.e., a fixed size of transmitted frames, to a general distribution of message sizes, provided this is the same for all nodes. The main modification on the model is a different Probability Density Function (PDF) of the virtual slot time.

having a fixed message size makes sense, in view of considering a given application run cooperatively by all nodes (e.g., CAM or event-driven notifications). Extension to heterogeneous nodes will be addressed in our future work.

B. Outline of access protocol

As soon as a new message is generated, the node starts the channel access procedure, eventually transmitting the message as a broadcast frame. When no one is transmitting[1], channel time is divided into back-off time slots. The duration of the back-off time slot is denoted with δ. A backlogged node selects a random back-off counter uniformly distributed between 1 and W_0, the contention window size. If the node senses an idle channel in a back-off time slot, it decrements its counter, otherwise it freezes the counter, waiting for the channel to become idle again for an Arbitration Inter-Frame Spacing (AIFS) time. When the counter hits 0, the node transmits. No acknowledgment is expected (and hence no re-transmission is envisaged), since only broadcast frames are used.

Handling of arriving messages is slightly different for the Node model and the Network model:

- In the Node model, new messages are generated according to a Poisson process with mean rate λ at each node. While engaged in the access procedure for a given message (contention plus transmission), a node only stores up to one further new message (the most recent), if any arrives. If more messages arrive, only the most recently arrived message is retained. This is consistent with modeling an application where only the freshest data is useful to feed the application process.
- In the Network model, new messages are generated according to a Bernoulli process with mean rate λ at each node. However, while engaged in the access procedure, a node cannot store any further messages. More precisely, new arriving messages can be accepted only when the current contention is resolved. As shown in Section V such a simplification does not undermine the precision of the model and avoids adding another dimension to the Markov chain.

The transmission time is assumed to be fixed, equal to T_0, e.g., all nodes use the same air bit rate and send messages having the same payload length. Relaxing this assumption to variable transmission times is possible, provided the message size probability distribution is same for all nodes. This impacts the duration of virtual slot times (the random variable X, see Equation (2)). Albeit tractable, math becomes quite involved, without any significant insight gain.

[1] More accurately, starting from when a fixed idle interval has expired, after the end of a transmission. In IEEE 802.11p MAC protocol, this fixed idle time is referred to as AIFS for the same traffic type, e.g. CAM messages, hence AIFS is a constant value [21] in this study, when it follows a data MAC PDU.

C. Notation and time evolution of the system

Time is divided into *virtual time slots*, each consisting of an idle back-off slot, or an idle back-off slot followed by a transmission. The first case occurs when nodes are either idle or count down to a positive back-off counter level. The latter case realizes when at least one back-off counter hits zero and hence the corresponding node starts a transmission.

For node j ($j = 1, \dots, n$) the following is specified

- $Q_j(t)$, the buffer content indicator of node j at time t. It can be either 0 or 1. If it is 1, it means that a message is waiting at the node while a frame carrying a previous message is under contention/transmission.
- $S_j(t)$, the MAC access layer state of node j. This state can be either idle, counting down or transmitting. Count down must further discriminate count down for the first transmission attempt after an idle period, regular count down and post-back-off count down.

The subscript j is dropped in the following, whenever there is no ambiguity.

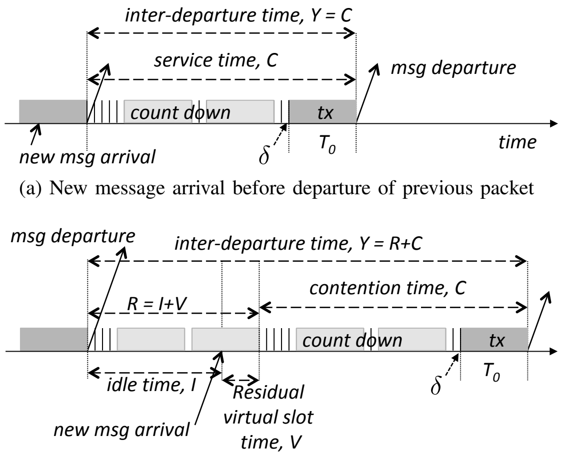
This state space scales with the number of stations, thus leading to excessively complex models. It will be shown that is it not necessary to keep track of all this information to predict the system performance with a good accuracy. Instead state information can be summarized according to two approaches:

- *Node model.* Considering only those embedded time points that correspond to the end of a transmission of a tagged node. Let t_k be the k-th embedded time point and let $x_k = x(t_k+)$ denote the value of the function $x(t)$ immediately after t_k. Only $Q_{1,k} = Q_1(t_k+)$ is tracked, where the tagged node is labelled as 1. All other nodes are accounted for by the average probability that they transmit in a virtual time slot (mean field approximation).
- *Network model.* Considering the process $i(t) = \sum_{j=1}^{n} Q_j(t)$, i.e., the overall number of backlogged stations (remember that for any single station $Q_j(t) = 0$ or 1 at any time t) and summarizing the MAC layer state in a virtual slot with the process $k(t)$ defined as the number of transmissions occurring at time t, $0 \le k(t) \le n$. This description leads to a bi-dimensional DTMC.

III. Node Model

According to our assumptions, the queue status $Q(t)$ of the tagged node can be either 0 (empty queue) or 1, in case there is a packet ready, while a previous packet is under contention or transmission.

Let t_k denote the time when the transmission of a packet is complete (k-th departing packet), and $Q_k = Q(t_k+)$ denotes the queue length left behind by the k-th departing packet. Let Y_k denote the packet inter-departure time between packet $k-1$ and k, i.e., $Y_k = t_k - t_{k-1}$. Let also C_k denote the service time of the k-th departing packet, defined as the time elapsing since the first idle back-off slot of the count down occurs until the packet transmission is completed (including the ensuing AIFS required for any other action on the wireless channel to be taken by any node).



(a) New message arrival before departure of previous packet

(b) New message arrival after departure of previous packet

Figure 1. Time evolution of the channel and definition of main time intervals of the Node Model.

Two situations can be distinguished by focusing on a tagged node (see Figure 1).

If there is already a packet ready to go upon the departure of the k-th packet, i.e., if $Q_k = 1$, contention starts immediately after packet departure. Therefore the time for the next packet to leave equals a service time C_{k+1}, i.e., $Y_{k+1} = C_{k+1}$ (upper time diagram in Figure 1a).

If instead the k-th departing packet leaves behind an empty queue, the node stays idle, until the next packet arrives (idle time I_{k+1}; see the lower time diagram of Figure 1b). In the meantime, the wireless channel is used by nodes other than the tagged one. The packet starting a new busy period has to wait for a residual virtual time slot, denoted with V_{k+1}. Then its contention starts. Overall, the inter-departure time is $Y_{k+1} = I_{k+1} + V_{k+1} + C_{k+1}$. In the following we denote $R_{k+1} = I_{k+1} + V_{k+1}$.

Summing up, we have

$$Y_{k+1} = \begin{cases} C_{k+1} & \text{if } Q_k = 1, \\ I_{k+1} + V_{k+1} + C_{k+1} & \text{if } Q_k = 0. \end{cases} \tag{1}$$

Given that there is no hidden node, the evolution of the time axis between transmission and idle time intervals is shared by all nodes, i.e., nodes are "synchronized". The time axis can be divided into *virtual time slots*, i.e., the time intervals elapsing between two successive idle back-off time slots[2]. A virtual time slot consists of just a single back-off time slot of duration δ, if no node transmits. On the contrary, if at least one node starts a transmission as soon as the empty back-off slots has gone by, the virtual time slot lasts $\delta + T_0$. In the following, the virtual time slot duration is denoted with X:

$$X = \begin{cases} \delta & \text{if no other node transmits ,} \\ \delta + T_0 & \text{otherwise.} \end{cases} \tag{2}$$

According to our assumptions, the sequence of virtual time slots forms a renewal process.

[2]The time axis is discretized this way also when all nodes are idle.

As for the service time, the tagged node decrements its back-off counter by one for each idle back-off time slots it sees on the channel. If the counter is denoted with K, the service time consists of $K-1$ virtual time slots and a final transmission time slot, where the tagged node transmits, possibly along with other nodes (collision). Therefore,

$$C = \sum_{j=1}^{K} X^{(j)} + \delta + T_0 \tag{3}$$

where K is a discrete random variable uniformly distributed in the set $\{1, \dots, W_0\}$.

The time from a departure of a packet that leaves an empty queue behind and the beginning of the contention time of the next arriving packet, namely R, is the sum of N consecutive virtual time slots:

$$R = \sum_{i=1}^{N} X^{(i)} \tag{4}$$

where N is a discrete random variable, defined as the number of virtual slots until a new packet arrives at the tagged node.

A. Node Model analysis

Starting from Equations (1) to (4), the Laplace transforms of the PDFs of X, C, R, V and Y can be derived. To proceed, we need the probability q that no other node transmits in a virtual time slot. Let τ denote the probability that a node transmits in a virtual time slot. Assuming that nodes other than the tagged one are independent of one another, we have:

$$q = (1-\tau)^{n-1} \tag{5}$$

How to compute τ will be provided later.

The Laplace transform of the PDF of X is (see Equation (2))

$$\varphi_X(s) = q e^{-\delta s} + (1-q) e^{-(T_0+\delta)s} \tag{6}$$

and the Laplace transform of the PDF of C is (see Equation (3))

$$\varphi_C(s) = e^{-(T_0+\delta)s} \frac{1-\varphi_X(s)^{W_0}}{W_0[1-\varphi_X(s)]}. \tag{7}$$

As for R, given the definition in Equation (4), conditional on $N=h$ and on $X^{(j)} = x_j$, $j = 1, \dots, h$, it is $R = x_1 + \dots, x_h$. Hence $\mathrm{E}[e^{-sR} | N = h, X^{(1)} = x_1, \dots, X^{(h)} = x_h] = \prod_{j=1}^{h} e^{-sx_j}$. Since arrivals occur according to a Poisson process of mean rate λ, the probability of $N = h$, conditional on $X^{(j)} = x_j$, $j = 1, \dots, h$ is $e^{-\lambda x_1} \dots e^{-\lambda x_{h-1}}(1 - e^{-\lambda x_h})$. Removing the conditioning we have:

$$\begin{aligned}
\varphi_R(s) &= \sum_{h=1}^{\infty} \int_0^{\infty} (1-e^{-\lambda x_h}) f_X(x_h) e^{-s x_h} dx_h \\
&\quad \times \prod_{j=1}^{h-1} \int_0^{\infty} e^{-\lambda x_j} f_X(x_j) e^{-s x_j} dx_j \\
&= \sum_{h=1}^{\infty} [\varphi_X(s+\lambda)]^{h-1} [\varphi_X(s) - \varphi_X(s+\lambda)] \\
&= \frac{\varphi_X(s) - \varphi_X(s+\lambda)}{1-\varphi_X(s+\lambda)}.
\end{aligned} \tag{8}$$

Given Equation (1), we find:

$$\varphi_Y(s) = \pi_0 \varphi_R(s) \varphi_C(s) + (1-\pi_0) \varphi_C(s) \tag{9}$$

where $\pi_0 = \mathcal{P}(Q = 0)$. An expression for π_0 is given in Equation (12).

Let us now turn to the evaluation of the Laplace transform of the PDF of V. It is the time elapsing since an arrival within a virtual time slot and the end of that virtual time slot.

Let A denote the random variable defined as the number of arrivals at the tagged node in a virtual time slot. Given $X = x$, it is $\mathcal{P}(A = k | X = x) = e^{-\lambda x} (\lambda x)^k / k!$, $k \ge 0$. Removing the conditioning, the generating function of A is $\phi_A(z) = \mathrm{E}[z^A] = \int_0^\infty \mathrm{E}[z^A | X = x] dF_X(x) = \varphi_X(\lambda - \lambda z)$.

Let us now define B as the number of arrivals given that at least one arrival occurs. We have

$$\begin{aligned}
b_k &= \mathcal{P}(B = k) = \mathcal{P}(A = k | A > 0) \\
&= \frac{\mathcal{P}(A = k, A > 0)}{\mathcal{P}(A > 0)} = \frac{a_k}{1-a_0}, \qquad k \ge 1.
\end{aligned}$$

and hence:

$$\phi_B(z) = \sum_{k=0}^{\infty} b_k z^k = \frac{\phi_A(z) - a_0}{1-a_0} = \frac{\varphi_X(\lambda - \lambda z) - a_0}{1-a_0} \tag{10}$$

The probability a_0 of no arrival in a virtual time slot is found simply as $a_0 = \phi_A(0) = \varphi_X(\lambda)$.

The arrivals occurring in V are just those following the first arrival in a virtual time slot, given that there is at least one arrival. Therefore, the generating function of the number of arrivals in V is given by $\phi_B(z)/z$. The same generating function can be expressed also as $\varphi_V(\lambda - \lambda z)$. Equating these two equivalent expressions, and letting $\lambda - \lambda z = s$, we get:

$$\begin{aligned}
\varphi_V(s) &= \left. \frac{\varphi_X(\lambda - \lambda z) - \varphi_X(\lambda)}{z[1-\varphi_X(\lambda)]} \right|_{z=1-s/\lambda} \\
&= \frac{\lambda}{\lambda - s} \frac{\varphi_X(s) - \varphi_X(\lambda)}{1-\varphi_X(\lambda)}.
\end{aligned} \tag{11}$$

Let $\pi_0 = \mathcal{P}(Q = 0)$ and $\pi_1 = \mathcal{P}(Q = 1)$. These probabilities can be found by using the transition probabilities between the two states $Q = 1$ and $Q = 0$, namely

$$\begin{aligned}
p_{10} &\equiv \mathcal{P}(Q_{k+1} = 0 | Q_k = 1) \\
&= \mathcal{P}(\text{no arrival in } C_{k+1}) = \varphi_C(\lambda) \\
p_{00} &\equiv \mathcal{P}(Q_{k+1} = 0 | Q_k = 0) \\
&= \mathcal{P}(\text{no arrival in } V_{k+1} + C_{k+1}) = \varphi_V(\lambda) \varphi_C(\lambda)
\end{aligned}$$

Using the equations above, the balance equations $\pi_0 = \pi_0 p_{00} + \pi_1 p_{10}$ and $\pi_0 + \pi_1 = 1$, we find

$$\pi_0 = \frac{\varphi_C(\lambda)}{1 + \varphi_C(\lambda) - \varphi_C(\lambda) \varphi_V(\lambda)} \tag{12}$$

To find τ, the renewal reward theorem is used. Let M be the number of virtual slots between two successive transmissions of the tagged node. We express τ as $\frac{1}{\mathrm{E}[M]}$. If the node goes idle upon a packet departure, it is $M = N + K$ (for the definitions of N and K see Equations (3) and (4)). If instead the departure leaves one packet behind, then it is $M = K$.

It is easy to check that N has a geometric probability distribution:

$$\mathcal{P}(N = h) = [1 - \varphi_X(\lambda)]\, \varphi_X(\lambda)^{h-1}\,,\ h \geq 1. \quad (13)$$

while K has a uniform probability distribution between 1 and W_0. The mean of M is therefore

$$\begin{aligned} \mathrm{E}[M] &= \pi_0\,(\mathrm{E}[N] + \mathrm{E}[K]) + (1 - \pi_0)\mathrm{E}[K] \\ &= \pi_0 \frac{1}{1 - \varphi_X(\lambda)} + \frac{W_0 + 1}{2} \end{aligned} \quad (14)$$

We have finally:

$$\tau = \frac{1}{\mathrm{E}[M]} = \frac{1}{\frac{2}{W_0+1} + \frac{\pi_0}{1-\varphi_X(\lambda)}} = \frac{\tau_0}{1 + \tau_0 \frac{\pi_0}{1-\varphi_X(\lambda)}} \quad (15)$$

where $\tau_0 = 2/(W_0 + 1)$ is the probability of transmission in a virtual slot in saturation. It can be checked that $\tau \to 0$ as $\lambda \to 0$ (light traffic regime), while $\tau \to \tau_0$ for $\lambda \to \infty$ (heavy traffic regime).

Note that τ is to be found by solving a fixed point equation, since the right hand side of Equation (15) depends on q, which in turn depends on τ. Since the relationship $\tau = F(\tau)$ in Equation (15) is a continuous function that maps the interval $[0, 1]$ onto itself, we can appeal to Brouwer's theorem to guarantee that the fixed point iteration converges.

B. *Performance metrics*

The conditional success probability γ of delivering a message, Packet Delivery Ratio (PDR), is

$$\gamma = (1 - \tau)^{n-1} \quad (16)$$

The AoI is the average age of messages received from other nodes at the tagged node. The mean AoI can be expressed as $\mathrm{E}[H] = \mathrm{E}[D^2]/(2\mathrm{E}[D])$, where D is the time between the *successful* reception of two consecutive messages from a given node. We have

$$D = \sum_{i=1}^{\ell} Y_i^{(i)} \quad (17)$$

where $\mathcal{P}(\ell = k) = (1 - \gamma)^{k-1}\gamma$ It is easy to find:

$$\mathrm{E}[H] = \frac{\mathrm{E}[Y^2]}{2\mathrm{E}[Y]} + \mathrm{E}[Y] \left(\frac{1}{P_s} - 1 \right) \quad (18)$$

The Channel Busy Ratio (CBR) is the average fraction of time that the channel is busy. It can be expressed as:

$$\rho = \frac{T_0}{\mathrm{E}[Y]} + \left(1 - \frac{T_0}{\mathrm{E}[Y]} \right) \frac{(1 - q)T_0}{\delta + (1 - q)T_0} \quad (19)$$

The normalized throughput Θ is the average number of successfully delivered messages per unit time divided by the offered rate of new messages. It is

$$\Theta = \frac{P_s/\mathrm{E}[Y]}{\lambda} \quad (20)$$

Note that Θ is less than 1 due to two sources of degradation: (i) message loss due to buffer overflow (only the most recent arriving message is maintained in node's buffer); (ii) message loss due to collisions on the wireless channel.

The coefficient of utilization of the channel is the average fraction of channel time used successfully to deliver messages. We have

$$U = \frac{T_0 P_s}{\mathrm{E}[Y]} = \frac{\tau(1 - \tau)^{n-1}}{\delta/T_0 + 1 - (1 - \tau)^n} \quad (21)$$

This last expression looks like the one holding for the saturation throughput of CSMA/CA, except that the saturation transmission probability $\tau_0 = 2/(W_0 + 1)$ is replaced with the transmission probability τ that is a function of λ.

Finally, the access delay is defined as the time since a new message arrival until when its transmission starts. It can be approximated by $\pi_0\mathrm{E}[V] + \mathrm{E}[C]$.

IV. Network Model

The starting point of the Network model (similarly to [22]) is the analysis of CSMA broadcasting proposed in [6] for unsaturated IEEE 802.11p network.

As in previous Section the channel access is operated according to a non-persistent CSMA protocol with immediate transmission, if the channel is seen as idle at the moment of packet arrival. The back-off procedure of a backlogged node is approximated with a probability of transmission in virtual time slot as $\tau_0 = 2/(W_0 + 1)$.

A. *Network Model analysis*

As explained in Section II-C the time evolution of a network is characterized by the embedded discrete-time two-dimensional Markov chain $\{i(t), k(t)\}$ with transitions occurring at the boundaries of virtual time slots and times of new messages acceptance, where

- $i(t)$ is the current number of nodes with packets. It can vary from 0 to n.
- $k(t)$ is the number of simultaneous transmissions in the CSMA channel (i.e. $k = 0$ means idle channel, $k = 1$ means ongoing successful transmission, $k > 1$ means ongoing collision).

To be consistent with the previous notations, new messages with transmission time T_0 are generated according to a Bernoulli process with mean rate λ at each node. In other words, a new message arrives to an inactive node with probability $\lambda\delta$ every back-off slot time δ.

The stationary probability distribution $\Omega(i, k)$ of this chain calculated for specific values of (λ, n) and the IEEE 802.11p network parameters T_0, δ and W_0 computed as explained in [6] is used to compute the performance metrics of interest.

The central part of the analysis is to calculate packet successful transmission probability P_s as follows:

$$P_s = 1 - \frac{\sum_{i=2}^{n}\sum_{k=2}^{i}\Omega(i, k)}{1 - \sum_{i=0}^{n}\Omega(i, 0)}. \quad (22)$$

The numerator of Equation (22) sums up the probabilities of channel states with two or more simultaneous packet transmissions, while the denominator is the probability that the channel is not idle.

B. Performance metrics

For PDR the following equation is used:

$$\gamma = \begin{cases} P_s & \text{if } 1/\lambda > n \cdot T_0 \\ P_s \cdot \frac{n \cdot T_0}{2 \cdot n \cdot T_0 - 1/\lambda} & \text{otherwise} \end{cases} \quad (23)$$

To obtain the AoI one can use the following approach:

$$\mathrm{E}[H] = \begin{cases} \frac{1}{\lambda} \cdot \frac{1}{P_s}, & \text{if } 1/\lambda > n \cdot T_0 \\ (n \cdot T_0 + \frac{n \cdot T_0 - \frac{W_0}{2} \cdot T_0}{2}) \cdot \frac{1}{P_s}, & \text{if } 1/\lambda < \frac{W_0}{2} \cdot T_0 \\ (n \cdot T_0 + \frac{n \cdot T_0 - \frac{1}{\lambda}}{2}) \cdot \frac{1}{P_s}, & \text{otherwise} \end{cases} \quad (24)$$

To calculate the CBR, one first needs to calculate the so-called collision factor, i.e., the average number of packets involved into a collision [22]:

$$c = \frac{\sum_{i=2}^{n} \sum_{k=2}^{i} k \cdot \Omega(i,k)}{\sum_{i=2}^{n} \sum_{k=2}^{i} \Omega(i,k)}. \quad (25)$$

Thus, the CBR can be obtained using (22) and (25) as:

$$\rho = \begin{cases} \lambda \cdot n \cdot T_0 \left[P_s + \frac{1}{c} \cdot (1 - P_s) \right], & \text{if } 1/\lambda > n \cdot T_0 \\ 1, & \text{otherwise} \end{cases} \quad (26)$$

where $\lambda \cdot n$ is total number of packets offered to the network per second, and the duration of a successful transmission takes time T_0 (to be more precise AIFS should be deducted in this specific equation). Collision factor c is used in (26) to account for the overlapping colliding packets.

Analogously with (24) the coefficient of utilization of the channel is

$$U = \begin{cases} \frac{n \cdot T_0}{1/\lambda} \cdot \frac{1}{P_s}, & \text{if } 1/\lambda > n \cdot T_0 \\ \frac{n \cdot T_0}{2 \cdot n \cdot T_0 - \frac{W_0}{2} \cdot T_0} \cdot \frac{1}{P_s}, & \text{if } 1/\lambda < \frac{W_0}{2} \cdot T_0 \\ \frac{n \cdot T_0}{2 \cdot n \cdot T_0 - \frac{1}{\lambda}} \cdot \frac{1}{P_s}, & \text{otherwise} \end{cases} \quad (27)$$

For the normalized packet throughput the following equation is used:

$$\Theta = P_s \cdot \min(1, \frac{1/\lambda}{n \cdot T_0}) \quad (28)$$

Finally, the access delay can be approximated by

$$\frac{W_0}{2} [\sum_{k=0}^{n} \sum_{i=0}^{k} \{ \Omega(i,k)(1 - \tau_0)^k (1 - \lambda\delta)^{n-k} \cdot \delta + (1 - (1 - \tau_0)^k (1 - \lambda\delta)^{n-k}) \cdot T_0 \}], \quad (29)$$

since each backoff slot (out of average total $W_0/2$) either has a duration of δ, when neither active node transmits nor any new packets arrive, or has a duration of T_0 for any transmission.

V. Performance Evaluation

The presented analytical models are validated against MATLAB® simulations implementing the IEEE 802.11p MAC layer (assuming an error free wireless channel) and consistent with the system model assumptions defined in Section II. Specifically, the time evolution of a node is characterized by a state (q, s), where

- q is the buffer content indicator; it can be either 0 or 1.
- s defines the MAC access state.

Table I
Simulation parameters.

Parameter	Value
Communication technology	IEEE 802.11p
Air bit rate	6 Mbit/s
Channel	Ideal (error free)
W_0	16
δ	0.013 ms
T_0	1.46 ms
Number of nodes n	10
Payload L	1000 Byte
Beacon interval *BI*	1–100 ms

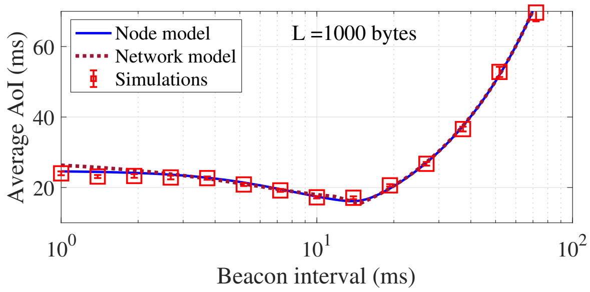


Figure 2. Average AoI as a function of the mean message inter-arrival time.

The performance metrics introduced in previous sections are plotted as a function of the mean inter-arrival time of new messages, which is referred to as the *beacon interval* in the following, i.e., *BI* $= 1/\lambda$. A range of message sending periods is considered which goes far beyond the currently envisaged limits in the context of vehicular networking standards, namely $1\,\text{ms} \leq BI \leq 100\,\text{ms}$. This is motivated by a twofold aim: (i) to push the system into saturation to check consistency of the models with known ones holding for saturated CSMA networks; (ii) to cover also future possible high precision applications, where high message frequency and very tight AoI requirements may be set. Other system parameters are illustrated in Table I.

In each plot the two models (Node and Network) are compared with simulations. Figure 2 plots the average AoI as a function of *BI*. Simulations correspond to square markers. Both models exhibit a remarkable accuracy. This will be confirmed also by all other plots. A steep increase is observed for large message sending periods, where the degradation of AoI is due to too sporadic updates. For very low sending periods there is essentially a saturation effect, which corresponds to CSMA network performance in a saturated regime. AoI is dominated by packet loss due to collision in this region. For intermediate values of the sending period, an optimum is stroked. The optimal message sending is found to be equal to n times the packet transmission time. This has been checked with many different values of n, ranging from $n = 2$ to several dozens of stations. Numerical results are not shown for space reasons.

The CBR is shown in Figure 3. The behavior of this curve highlights the *phase-transition* phenomenon characterizing the performance of the CSMA network. For large message

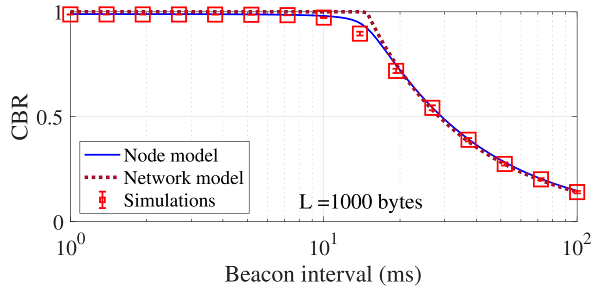


Figure 3. CBR as a function of the mean message inter-arrival time.

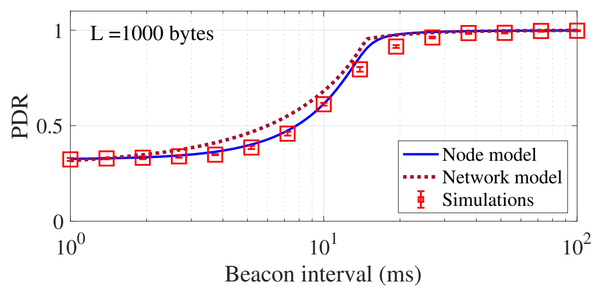


Figure 4. Probability of a successful packet delivery as a function of the mean message inter-arrival time.

generation periods the CBR is low, while it saturates quickly to almost 100 % as messages are sent faster and faster. CBR is used as feedback to control the network load in vehicular ad hoc networks through decentralized congestion control algorithms. In a real-world deployment of IEEE 802.11p, a CBR above 62 % will force nodes to restrict the number of messages transmitted on the shared communication channel [23].

Figure 4 plots the PDR as a function of *BI*. Here too the phase-transition behavior is evident. It is noted that the PDR gets quite low when the network is pushed into saturation, while it is still less than 1 around the optimal working point where AoI is minimal (corresponding to the transition region between the saturation regime and the light load regime, where PDR ≈ 1).

The utilization[3] plot in Figure 5 is quite interesting. It has a

[3]In the "classic" literature on CSMA models this is often called (saturation) throughput.

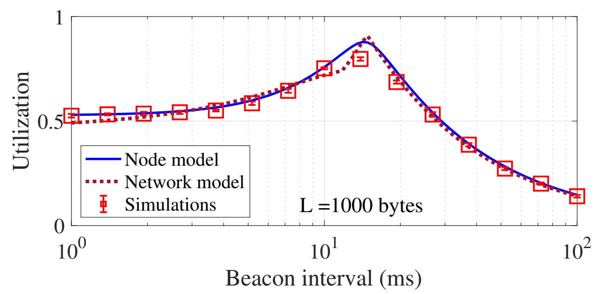


Figure 5. Average channel utilization coefficient as a function of the mean message inter-arrival time.

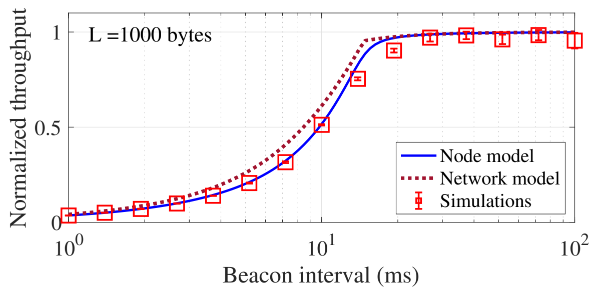


Figure 6. Average normalized packet throughput as a function of the mean message inter-arrival time.

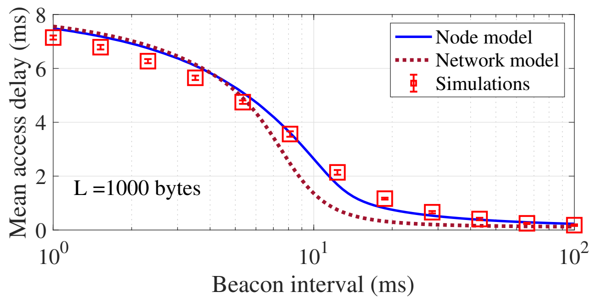


Figure 7. Mean access delay as a function of the mean message inter-arrival time.

marked peak corresponding to the optimal operating point of the system (the one minimizing AoI). The presence of a spike of the utilization, where the fraction of time that the channel is used successfully overshoots beyond the value it settles on in saturation, is a known since Bianchi's paper [7]. We offer here a model able to correctly predict this behavior.

Figure 6 shows the normalized throughput Θ as a function of *BI*. This throughput measures the fraction of generated messages that get through the network successfully. A very low throughput is achieved in saturation, due to collisions and even more to buffer overflows (newly arriving messages overwhelm the node buffers, due to the slowdown of message sending that nodes incur in saturation). The high throughput registered with low load levels means that essentially all messages get through (low probability of buffer overflow, low collision rate in the wireless channel.) It is therefore a measure of the accuracy with which we manage to transfer information, but a high throughput does not necessarily corresponds to timely information. In fact, it appears that high throughput is not equivalent to low AoI, which is a well known fact.

Figure 7 shows that IEEE 802.11p supports very low channel access delay – the delay is below 1 ms for a CBR around 62 %, see Figure 3. As expected, the access delay increases when the network is pushed into saturation.

VI. Conclusion and Outlook

We have presented two analytical models for a fully connected IEEE 802.11p network where nodes exchange broadcast update messages. Our performance analysis shows that the models are efficient and accurate. They allow the evaluation of

AoI and several other metrics. The models also offer insight to optimize the message generation rate so as to minimize the AoI and maximize the utilization of the wireless channel. Generalizations of the models are possible, e.g., considering variable payload length of messages, more general arrival patterns with respect to Poisson arrivals (e.g., Markovian Arrival Process).

The analytical results show that IEEE 802.11p supports a low channel access delay and high freshness of information concurrently, which are two important metrics for supporting advanced connected and automated driving use cases. In a real-world deployment, the vehicular ad hoc networking avoids saturated networks through exercising congestion control. This implies that the number of messages introduced to the network by each node is restricted. The congestion control will be activated in scenarios with many vehicles within radio range, which occurs for example in a traffic jam when vehicles are moving slowly (or standing still). Then the need for channel resources is low due to low vehicle dynamics and, thus, information exchange is not needed that frequently.

Future work includes the extension of these models to the much more difficult case of extended network of nodes, where partial sensing is possible (hidden nodes), as in the case of vehicular networks in urban environments. Analysis of partial sensing, comparison with more realistic simulations that include micro-mobility of vehicles, as well as wireless channel and physical layer characteristics, is another target of future work.

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